BRIDGING MATHEMATICS STUDENTS: ATTITUDES, AUTONOMOUS LEARNING BEHAVIOURS, AND PROBLEM SOLVING

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This paper describes a pilot study for a research project investigating aspects of autonomous learning behaviours, attitudes towards mathematics, and problem solving performance of students enrolled in a year-long bridging mathematics course. Six attitudinal variables were selected for investigation. A problem solving exercise was used as the context for investigating three autonomous learning behaviours. Further directions for research are discussed.

INTRODUCTION

The Study Context

Subjects. In recent years, students whose mathematics achievement level is not high enough to enter university science courses, have been given increased opportunities to attain the mathematics level required, by completing a bridging course in mathematics. These courses range from intensive short 4-6 week courses, to "Year 12 equivalent" studies taking a whole academic year. Such courses not only provide "catch-up" opportunities for recent school leavers, but also provide opportunities for mature-age people to return to study.

The students in this project were enrolled in a year-long Bridging Mathematics course, designed to meet requirements for entry into first year mathematics courses. Most of the students were older students returning to study.

Background to the study. It has been proposed (Fennema and Peterson,1985) that autonomous learning behaviours (such as independent learning, perseverance, taking risks, and willingness to choose a challenging task) are necessary in higher levels of mathematics; that these behaviours develop over time, and are related to the student's internal belief system, as well as external/societal factors. The student's belief system includes attitudes towards mathematics. Attitudes that Fennema and Peterson (1985) found to be important were confidence in learning mathematics; causal attributions and perceived usefulness of mathematics.

In secondary school mathematics students' confidence has been found to be a good predictor of future mathematics achievement and participation (Eccles and Jacobs,1986; Rowe,1988). Causal attribution patterns in secondary school mathematics students have also been proposed as important (Wolleat, Pedro, Becker and Fennema,1980). Elliott (1990) found that these attitudes, as well as the student's perceived usefulness of mathematics, were also important in the mathematics achievement of tertiary students. Hence these three attitudes were chosen for investigation in this study.

Many Bridging Mathematics students, who have enrolled in the course in order to achieve entry into the tertiary course of their choice, appear to be highly motivated. Many also show signs of anxiety, especially at the start of the course. Hence the attitudes of intrinsic motivation, maths anxiety, and attitude towards success in mathematics were also selected for investigation.

In recent years there has been an increasing emphasis on problem solving in mathematics courses. Problem solving, rather than only straightforward applications of algorithms, has become an important objective in the learning of mathematics. The student's belief system and its influence on problem solving performance, was cited by Silver (1985) as an area of research which could make a valuable contribution to the understanding of how mathematics is learnt. Problem solving performance was incorporated into this study as a measure of the level of mathematical thinking of the students, and because of the influence of affective variables in problem solving (McLeod,1989).

Research questions

This project was a pilot study investigating aspects of autonomous learning behaviours, attitudes towards mathematics, and problem solving performance of Bridging Mathematics students. The research questions were:

1. What attitudes towards mathematics are held by students entering the Bridging Mathematics course? How do these attitudes change during the year?

   Specific attitudes:
   - confidence in learning and doing mathematics
   - motivation
   - perceived usefulness of mathematics
   - mathematics anxiety
   - attitude towards success in mathematics
   - causal attributions in mathematics

2. What evidence of autonomous learning behaviours is shown by Bridging Mathematics students?

   Specific autonomous learning behaviours:
   - independent learning
   - perserverance
   - choosing to work on a challenging task

3. How does problem solving performance develop during the year?

METHOD

Attitudes

The testing instrument used was an Attitudes Questionnaire, consisting of two parts.

Part A was based on the Fennema-Sherman Mathematics Attitude Scales (1976) and consisted of forty (40) statements, using the five-point Likert scale. It contained five (5) subscales, which were:

confidence in learning mathematics
perceived usefulness of mathematics
motivation
mathematics anxiety
attitude towards success in mathematics

Part B investigated causal attributions: whether success/failure in mathematics was attributed to ability level; amount of effort; ease or difficulty of task; or environmental factors such as luck or teacher input. This part of the questionnaire was based on the Mathematics Attribution Scale (MAS) (Fennema, Wolleat and Pedro,1979).

The questionnaire was administered three times. It was administered to eighteen (18) Bridging Mathematics students at the beginning of Semester 1. It was administered to fourteen (14) of these students at the beginning of Semester 2, and at the end of Semester 2.

Of the initial eighteen (18) subjects, three (3) were female and fifteen (15) were male. Five (5) were less than 21 years old, and thirteen (13) were over 21 years of age. Of the fourteen (14) students who completed the year, two (2) were female and twelve (12) were male. Three (3) were less than 21 years old, and eleven (11) were over 21 years of age.

Autonomous learning behaviours

The main investigation of autonomous learning behaviours was a Problem Solving Exercise completed by five (5) Bridging Mathematics students towards the end of Semester 2. The subjects were asked to rank a set of eight (8) problems from easiest (1) to hardest (8). Each subject then selected three (3) problems to attempt. The problems chosen by the subject indicated the subject's willingness to work on tasks perceived to be challenging.

For each problem attempted, the subject was asked to show all working on the solution; keep a record of the time taken, and answer a questionnaire relating to the problem after it had been attempted. The time spent on a difficult problem indicated level of perserverance, and the extent to which other help was sought suggested level of independence.

Problem solving

An initial set of six (6) problems was given to the Bridging Mathematics students at the beginning of Semester 1. The problems were multiple-choice, but students were asked to record all working out of the solutions.

The Problem Solving Exercise (which involved autonomous learning behaviours, as described above) was administered towards the end of Semester 2, to ascertain the problem solving development of the five (5) students tested.

RESULTS

Attitudes

Questionnaire: Part A. The mean scores for motivation, attitude to success, and perceived usefulness of mathematics were high for Bridging Mathematics students at the start of Semester 1. Anxiety was also high, and confidence ranged from high to very low. When students were separated by gender, it was found that females had a much lower confidence level than did males (Mann-Whitney U 15,3 = 4.5, $p<.025$); a much higher anxiety level (U 15,3 = 2.5, $p<.005$), and a stronger attitude towards success (U 15,3 = 2.5, $p<.005$)

When students were separated according to whether they had left the course by the end of Semester 1, or had completed the year, it was found that those who left had less confidence (Mann-Whitney U 14,4 = 13, $p<.1$), less motivation (U 14,4 = 7.5, $p<.025$), and did not see mathematics as useful as those who stayed (U 14,4 = 8, $p<.025$)

An analysis of the questionnaire for the fourteen (14) students who completed the course, showed that the only attitude which changed significantly over the year was mathematics anxiety. (ANOVA, F = 6.227, $p<.01$). A post hoc test (Tukey's HSD = 3.34, $p<.05$ for u1 = u2) supported the conclusion that anxiety decreased significantly during Semester 1, and then did not change in level.

Questionnaire: Part B. The results of the attribution scores of the fourteen (14) students tested three (3) times over the year, showed that the class, on the whole attributed success in mathematics to effort and environmental factors. Failure in mathematics was attributed largely to lack of effort. There was no significant change in the scores for these attributions over the year.

Overall, only two attributional variables changed significantly during the year. Attributing failure to difficulty of task decreased over Semester 1, but not over the whole year (ANOVA, F = 2.667, $p<.01$; post hoc test Tukey's HSD = .265, $p<.05$ for u1 = u2). Attributing failure to environmental factors decreased over Semester 1, and then showed no further change (ANOVA, F = 5.271, $p<.01$; post hoc test Tukey's HSD = .486, $p<.05$ for u1 = u2).

Autonomous learning behaviours

The Problem Solving Exercise given to five (5) students during Semester 2, provided much qualitative data concerning the three (3) autonomous learning behaviour variables selected. Table 1 contains a summary of these results. The students showed evidence of autonomous learning behaviours, especially independence and perseverance.

Table 1: Autonomous Learning Behaviours

Subject	Independent learning	Choosing a challenge	Perserverance
1	worked independently	Ranks:1,2,3 Problems chosen for ease of task.1 problem chosen for interest	Completed 2 problems quickly. Spent 35 mins. on the third; did not complete.
2	worked independently; referred to notes.	Ranks:1,2,3 2 problems chosen for ease of task; 1 chosen for interest,challenge.	Completed 2 problems in short time. Persisted to completion the third problem (56 min)
3	worked independently	Ranks: 1,2,6 Chose 1 problem for ease of task; 1 for interest,1 for challenge	Completed 1 problem quickly. Persisted to completion 2 problems returning to each several times (24 min , 100 min)
4	worked independently	Ranks:2,4,5 Chose 1 problem for ease of task, 1 for interest,1 for challenge	Completed 2 problems quickly. Persisted with third problem for several hours (not completed)
5	worked independently; looked up formula	Ranks: 6,7,8 All problems chosen for interest , challenge	Persisted with all 3 problems; 2 completed (21 min , 30 min) ; 1 incomplete (150 min in 7 sessions)

Problem solving performance

Initial problem solving exercise. The mean number of correct answers to the set of six problems given to the Bridging Mathematics students at the start of Semester 1 was 4.47 (s.d. = 1.46)

The written solutions were analysed using a schema constructed by Charles and Lester (1982). A score of 0,1 or 2 was given to each of three aspects of each solution, depending on the completeness or correctness of the aspect. The three aspects were:

1. Understanding the problem (mean = 1.76; s.d. = 0.243)
2. Solving the problem (mean = 1.63; s.d. = 0.303)
3. Answering the problem (mean = 1.52; s.d. = 0.458)

The students mostly understood the problems, and gave at least partial solutions and answers, but the range of problem solving performance was greater for the latter two aspects.

Later problem solving exercise. The solutions to the problems attempted by the five students who did the Problem Solving Exercise in Semester 2 were analysed using the schema described above. The results of that analysis, and the results of the analysis of the first set of problems done by the same students, are found in Table 2.

Table 2: Mean Scores for Initial and Later Problem Solving Exercises

Subject	Exercise	No. Problems Correct	Understand -ing (0 to 2)	Solution (0 to 2)	Answer (0 to 2)
1	Initial	6	2	2	2
	Later	1	1.67	1.33	1.33
2	Initial	3	1.5	1.33	1.17
	Later	1	2	1.33	1.33
3	Initial	6	1.83	1.83	2
	Later	2	2	1.33	1
4	Initial	6	1.83	1.83	2
	Later	2	2	1.33	1
5	Initial	6	2	2	2
	Later	1	2	1.76	1.33

The number of problems in the first exercise was six (6), and for the later exercise was three (3).

Table 2 shows that students got proportionally less correct solutions for the later exercise, while still showing a high level of understanding of the problems set. It should be noted that the first set of problems were multiple-choice, so that students could have chosen the answer by elimination,or by guessing. The written solutions for the later exercise in fact showed that the students had considerable problem solving skills.

DISCUSSION

Attitudes

Results from the three Attitude Questionnaire tests were largely as expected from the literature and observation. Students were highly motivated, perceived mathematics to be very useful, and had a strong attitude towards success. These attitudes remained high for the whole year. Confidence levels were widely spread, with a fairly high mean, and remained so for the whole course. Mathematics anxiety, which also varied widely amongst the students, decreased during Semester 1, but did not change further.

During the Bridging Mathematics course, students consistently attributed success in mathematics to their own efforts, and to environmental factors, rather than to their own ability, or ease of task set. Failure in mathematics was attributed to lack of effort. During Semester 1, students became less likely to attribute failure to difficulty of task, or environmental factors. Overall, the students attributed success or failure to their own efforts, but did not seem to recognize their mathematics ability.

Autonomous learning behaviours

The students who participated in the Problem Solving Exercise showed evidence of autonomous learning behaviours. All worked independently, and most showed a high level of perserverance. Four chose at least one problem as a challenge.

All the participants were in the upper half of the class for mathematics achievement. The analysis would have been more valuable if less capable students had also participated in the exercise. A further direction for research would be to set the exercise earlier in the year, as well, so as to investigate the development of autonomous behaviours.

Problem solving

The solutions produced by the students in the Problem Solving Exercise in Semester 2, were rich in qualitative data, which was not adequately analysed by the schema used. A deeper analysis of the solutions presented could give more information about the problem solving performance of the students.

Since the initial problem solving exercise in Semester 1 contained multiple-choice questions, the resulting worked solutions could not be properly compared with the later set of problems done in Semester 2. In further research, the initial set of problems should not be multiple-choice, thus allowing for a clearer study of the development of problem solving performance.

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